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Interface of General Relativity, Quantum Physics and Statistical Mechanics: Some Recent Developments

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Abstract

The arena normally used in black holes thermodynamics was recently generalized to incorporate a broad class of physically interesting situations. The key idea is to replace the notion of stationary event horizons by that of 'isolated horizons.' Unlike event horizons, isolated horizons can be located in a space-time *quasi-locally*. Furthermore, they need not be Killing horizons. In particular, a space-time representing a black hole which is itself in equilibrium, but whose exterior contains radiation, admits an isolated horizon. In spite of this generality, the zeroth and first laws of black hole mechanics extend to isolated horizons. Furthermore, by carrying out a systematic, non-perturbative quantization, one can explore the quantum geometry of isolated horizons and account for their entropy from statistical mechanical considerations. After a general introduction to black hole thermodynamics as a whole, these recent developments are briefly summarized.

I. MOTIVATION

In the seventies, there was a flurry of activity in black hole physics which brought out an unexpected interplay between general relativity, quantum field theory and statistical mechanics [1–4]. That analysis was carried out only in the semi-classical approximation, i.e., either in the framework of Lorentzian quantum field theories in curved space-times or by keeping just the leading order, zero-loop terms in Euclidean quantum gravity. Nonetheless, since it brought together the three pillars of fundamental physics, it is widely believed that these results capture an essential aspect of the more fundamental description of Nature. For over twenty years, a concrete challenge to all candidate quantum theories of gravity has been to derive these results from first principles, without invoking semi-classical approximations.

Specifically, the early work is based on a somewhat ad-hoc mixture of classical and semi-classical ideas —reminiscent of the Bohr model of the atom— and generally ignored the quantum nature of the gravitational field itself. For example, statistical mechanical parameters were associated with macroscopic black holes as follows. The laws of black hole mechanics were first derived in the framework of *classical* general relativity, without any reference to the Planck's constant $\hbar$ [2]. It was then noted that they have a remarkable similarity with the laws of thermodynamics if one identifies a multiple of the surface gravity κ of the black hole with temperature and a corresponding multiple of the area $a_{\rm hor}$ of its horizon with entropy. However, simple dimensional considerations and thought experiments showed that the multiples must involve $\hbar$, making quantum considerations indispensable for a fundamental understanding of the relation between black hole mechanics and thermodynamics [1]. Subsequently, Hawking's investigation of (test) quantum fields propagating on a black hole geometry showed that black holes emit thermal radiation at temperature $T_{\rm rad} = \hbar\kappa/2\pi$ [3]. It therefore seemed natural to assume that black holes themselves are hot and their temperature $T_{\rm bh}$ is the same as $T_{\rm rad}$. The similarity between the two sets of laws then naturally suggested that one associate an entropy $S_{\rm bh} = a_{\rm hor}/4\hbar$ with a black hole of area $a_{\rm hor}$. While this procedure seems very reasonable, it does not provide a 'fundamental derivation' of the thermodynamic parameters $T_{\rm bh}$ and $S_{\rm bh}$. The challenge is to derive these formulas from first principles, i.e., by regarding large black holes as statistical mechanical systems in a suitable quantum gravity framework.

Recall the situation in familiar statistical mechanical systems such as a gas, a magnet or a black body. To calculate their thermodynamic parameters such as entropy, one has to first identify the elementary building blocks that constitute the system. For a gas, these are molecules; for a magnet, elementary spins; for the radiation field in a black body, photons. What are the analogous building blocks for black holes? They can not be gravitons because the underlying space-times were assumed to be stationary. Therefore, the elementary constituents must be non-perturbative in the terminology of local field theory. Thus, to account for entropy from first principles within a candidate quantum gravity theory, one would have to: i) isolate these constituents; ii) show that, for large black holes, the number of quantum states of these constituents goes as the exponential of the area of the event horizon; and, iii) account for the Hawking radiation in terms of processes involving these constituents and matter quanta.

These are difficult tasks, particularly because the very first step –isolating the relevant constituents– requires new conceptual as well as mathematical inputs. Furthermore, in the semi-classical theory, thermodynamic properties have been associated not only with black holes but also with cosmological horizons. Therefore, ideally, the framework has to be sufficiently general to encompass these diverse situations. It is only recently, more than twenty years after the initial flurry of activity, that detailed proposals have emerged. The more well-known of these comes from string theory [28] where the relevant elementary constituents are associated with D-branes which lie outside the original perturbative sector of the theory. The purpose of this contribution is to summarize the ideas and results from another approach which emphasizes the quantum nature of geometry, using non-perturbative techniques from the very beginning. Here, the elementary constituents are the quantum excitations of geometry itself and the Hawking process now corresponds to the conversion of the quanta of geometry to quanta of matter. Although the two approaches seem to be strikingly different from one another, as I will indicate, in a certain sense they are complementary.

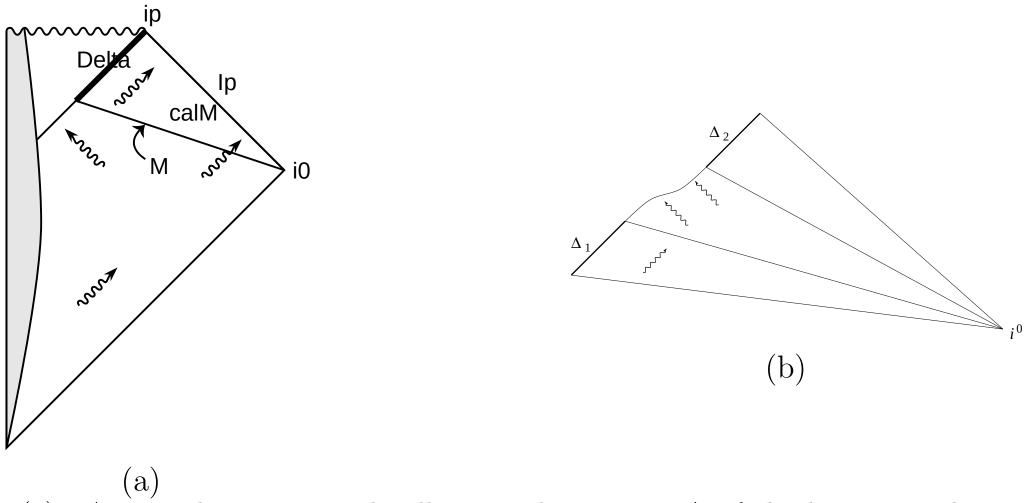

FIG. 1. (a) A typical gravitational collapse. The portion Δ of the horizon at late times is isolated. The space-time $\mathcal{M}$ of interest is the triangular region bounded by Δ, $\mathcal{I}^+$ and a partial Cauchy slice M. (b) Space-time diagram of a black hole which is initially in equilibrium, absorbs a small amount of radiation, and again settles down to equilibrium. Portions Δ_1 and Δ_2 of the horizon are isolated.

II. KEY ISSUES

In the last section, I focussed on quantum issues. However, the status of *classical* black hole mechanics, which provided much of the inspiration in quantum considerations, has itself remained unsatisfactory in some ways. Therefore, in a systematic approach, one has to revisit the classical theory before embarking on quantization.

The zeroth and first laws of black hole mechanics refer to equilibrium situations and small departures therefrom. Therefore, in this context, it is natural to focus on isolated black holes. However, in standard treatments, these are generally represented by *stationary* solutions of field equations, i.e, solutions which admit a time-translation Killing vector field *everywhere*, not just in a small neighborhood of the black hole. While this simple idealization is a natural starting point, it seems to be overly restrictive. Physically, it should be sufficient to impose boundary conditions at the horizon which ensure *only the black hole itself is isolated.* That is, it should suffice to demand only that the intrinsic geometry of the horizon be time independent, whereas the geometry outside may be dynamical and admit gravitational and other radiation. Indeed, we adopt a similar viewpoint in ordinary thermodynamics; in the standard description of equilibrium configurations of systems such as a classical gas, one usually assumes that only the system under consideration is in equilibrium and stationary, not the whole world. For black holes, in realistic situations one is typically interested in the final stages of collapse where the black hole is formed and has ‘settled down’ or in situations in which an already formed black hole is isolated for the duration of the experiment (see figure 1). In such situations, there is likely to be gravitational radiation and non-stationary matter far away from the black hole, whence the space-time as a whole is not expected to be stationary. Surely, black hole mechanics should incorporate in such situations.

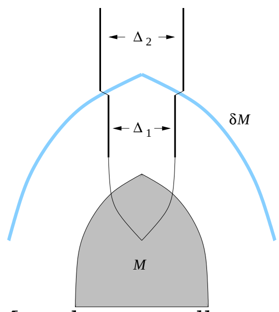

FIG. 2. A spherical star of mass M undergoes collapse. Later, a spherical shell of mass δM falls into the resulting black hole. While Δ_1 and Δ_2 are both isolated horizons, only Δ_2 is part of the event horizon.

A second limitation of the standard framework lies in its dependence on *event* horizons which can only be constructed retroactively, after knowing the *complete* evolution of space-time. Consider for example, Figure 2 in which a spherical star of mass M undergoes a gravitational collapse. The singularity is hidden inside the null surface Δ_1 at $r = 2M$ which is foliated by a family of marginally trapped surfaces and would be a part of the event horizon if nothing further happens. Suppose instead, after a very long time, a thin spherical shell of mass δM collapses. Then Δ_1 would not be a part of the event horizon which would actually lie slightly outside Δ_1 and coincide with the surface $r = 2(M + \delta M)$ in distant future. On physical grounds, it seems unreasonable to exclude Δ_1 a priori from thermodynamical considerations. Surely one should be able to establish the standard laws of laws of mechanics not only for the event horizon but also for Δ_1.

Another example is provided by cosmological horizons in de Sitter space-time [4]. In this case, there are no singularities or black-hole event horizons. On the other hand, semi-classical considerations enable one to assign entropy and temperature to these horizons as well. This suggests the notion of event horizons is too restrictive for thermodynamical analogies. We will see that this is indeed the case; as far as equilibrium properties are concerned, the notion of event horizons can be replaced by a more general, quasi-local notion of 'isolated horizons' for which the familiar laws continue to hold. The surface Δ_1 in figure 2 as well as the cosmological horizons in de Sitter space-times are examples of isolated horizons.

At first sight, it may appear that only a small extension of the standard framework, based on stationary event horizons, is needed to overcome the limitations discussed above. However, this is not the case. For example, in the stationary context, one identifies the black-hole mass with the ADM mass defined at spatial infinity. In the presence of radiation, this simple strategy is no longer viable since radiation fields well outside the horizon also contribute to the ADM mass. Hence, to formulate the first law, a new definition of the black hole mass is needed. Similarly, in the absence of a global Killing field, the notion of surface gravity has to be extended in a non-trivial fashion. Indeed, even if space-time happens to be static in a neighborhood of the horizon —already a stronger condition than contemplated above— the notion of surface gravity is ambiguous because the standard expression fails to be invariant under constant rescalings of the Killing field. When a *global* Killing field exists, the ambiguity is removed by requiring the Killing field be unit at *infinity*.

Thus, contrary to intuitive expectation, the standard notion of the surface gravity of a stationary black hole refers not just to the structure at the horizon, but also to infinity. This 'normalization problem' in the definition of the surface gravity seems especially difficult in the case of cosmological horizons in (Lorentzian) space-times whose Cauchy surfaces are compact. Apart from these conceptual problems, a host of technical issues must also be resolved. In Einstein-Maxwell theory, the space of stationary black hole solutions is three dimensional whereas the space of solutions admitting isolated horizons is *infinite*-dimensional since these solutions admit radiation near infinity. As a result, new techniques have to be used and these involve some functional analytic subtleties.

This set of issues has a direct bearing on quantization as well. For, in a systematic approach, one would first extract an appropriate sector of the theory in which space-time geometries satisfy suitable conditions at interior boundaries representing horizons, then introduce a well-defined action principle tailored to these boundary conditions, and, finally, use the resulting Lagrangian or Hamiltonian frameworks as points of departure for constructing the quantum theory. If one insists on using *event* horizons, these steps are difficult to carry out because the resulting boundary conditions do not translate in to (quasi-)local restrictions on fields. Indeed, for event horizon boundaries, there is *no* action principle available in the literature. The restriction to *globally* stationary space-times causes additional difficulties. For, by no hair theorems, the space of stationary solutions admitting event horizons is finite dimensional and quantization of this 'mini-superspace' would ignore all field theoretic effects *by fiat*. Indeed, most treatments of black hole mechanics are based on differential geometric identities and field equations, and are not at all concerned with such issues related to quantization.

Thus, the first challenge is to find a new framework which achieves, in a single stroke, three goals: i) it overcomes the two limitations of black hole mechanics by finding a better substitute for stationary event horizons; ii) generalizes laws of black hole mechanics to the new, more physical paradigm; and, iii) leads to a well-defined action principle and Hamiltonian framework which can serve as spring-boards for quantization. The second challenge is then to: i) carry out quantization non-perturbatively; ii) obtain a quantum description of the horizon geometry; and, iii) account for the the horizon entropy statistical mechanically by counting the underlying micro-states. As discussed in the next section, these goals have been met for non-rotating isolated horizons.

III. SUMMARY

In this section, I will sketch the main ideas and results on the classical and quantum physics of isolated horizons and provide a guide to the literature where details can be found.

A. Isolated horizons

The detailed boundary conditions defining non-rotating isolated horizons were introduced in [11,13]. Basically, an isolated horizon Δ is a null 3-surface, topologically $S^2 \times R$, foliated by a family of marginally trapped 2-spheres. Denote the normal direction field to Δ by $[\ell^a]$.

Being null, it is also tangential to Δ. The boundary conditions require that it be expansion-free, so that the area of the marginally trapped surface remains constant 'in time'. Assuming that the matter fields under consideration satisfy a very weak 'energy condition' at Δ, the Raychaudhuri equation then implies that there is no flux of matter across Δ. More detailed analysis also shows that there is no flux of gravitational radiation. (More precisely, the Newman-Penrose curvature component Ψ_0 vanishes on Δ.) These properties capture the idea that the horizon is isolated. Denote the second null normal to the family of marginally trapped 2-spheres by $[n^a]$. There are additional conditions on the Newman-Penrose spin coefficients associated with $[n^a]$ which ensure that Δ is a *future* horizon with no rotation.

Event horizons of static black holes of the Einstein-Maxwell-Dilaton theory are particular examples of non-rotating isolated horizons. The cosmological horizons in de Sitter space-time provide other examples. However, there are many other examples as well; the space of solutions admitting isolated horizons is in fact *infinite* dimensional [15,13].

All conditions in the definition are *local* to Δ whence the isolated horizon can be located quasi-locally; unlike the event horizon, one does not have to know the *entire* space-time to determine whether or not a given null surface is an isolated horizon. Also, there may be gravitational or other radiation arbitrarily close to Δ. [1]Therefore, in general, space-times admitting isolated horizons need not be stationary *even in a neighborhood of* Δ; isolated horizons need not be Killing horizons [15]. In spite of this generality, the intrinsic geometry, several of the curvature components and several components of the Maxwell field at any isolated horizon are the same as those at the event horizon of Reissner-Nordström space-times [13,11,14]. This similarity greatly simplifies the detailed analysis.

Finally, isolated horizons are special cases of Hayward's trapping horizons [19], the most important restriction being that the direction field $[\ell^a]$ is assumed to be expansion-free. Physically, as explained above, this restriction captures the idea that the horizon is 'isolated', i.e., we are dealing with an equilibrium situation. The restriction also gives rise to some mathematical simplifications which, in turn, make it possible to introduce a well-defined action principle and Hamiltonian framework. As we will see below, these structures play an essential role in the proof of the generalized first law and in passage to quantization.

B. Mechanics

Let me begin by placing the present work on mechanics of isolated horizons in the context of other treatments in the literature. The first treatments of the zeroth and first laws were given by Bardeen, Carter and Hawking [2] for black holes surrounded by rings of perfect fluid and this treatment was subsequently generalized to include other matter sources [5]. In all these works, one restricted oneself to globally stationary space-times admitting event horizons and considered transitions from one such space-time to a nearby one. Another approach, based on Noether charges, was introduced by Wald and collaborators [20,6]. Here,

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[1]During this conference Piotr Chruciel pointed out that the Robinson-Trautman solutions provide examples of exact solutions which admit non-rotating isolated horizons, have no Killing fields and admit radiation arbitrarily close to the horizon.

one again considers stationary event horizons but allows the variations to be arbitrary. Furthermore, this method is applicable not only for general relativity but for stationary black holes in a large class of theories. In both approaches, the surface gravity κ and the mass M of the hole were defined using the global Killing field and referred to structure at infinity.

The zeroth and first laws were generalized to arbitrary, non-rotating isolated horizons Δ in the Einstein-Maxwell theory in [12,13] and dilatonic couplings were incorporated in [14]. In this work, the surface gravity κ and the mass M_Δ of the isolated horizon refer only to structures *local* to Δ.[2] As mentioned in section III A, the space $\mathcal{IH}$ of solutions admitting isolated horizons is infinite dimensional and static solutions constitute only a finite dimensional sub-space $\mathcal{S}$ of $\mathcal{IH}$. Let us restrict ourselves to the non-rotating case for comparison. Then, in treatments based on the Bardeen-Carter-Hawking approach, one restricts oneself only to $\mathcal{S}$ and variations tangential to $\mathcal{S}$. In the Wald approach, one again restricts oneself to points of $\mathcal{S}$ but the variations need not be tangential to $\mathcal{S}$. In the present approach, on the other hand, the laws hold at *any* point of $\mathcal{IH}$ and *any* tangent vector at that point. However, so far, our results pertain only to *non-rotating* horizons in a restricted class of theories.

The key ideas in the present work can be summarized as follows. It is clear from the setup that surface gravity should be related to the acceleration of $[\ell^a]$. Recall, however, the acceleration is not a property of a direction field but of a vector field. Therefore, to define surface gravity, we must pick out a specific vector field ℓ^a from the equivalence class $[\ell^a]$. Now, the shear, the twist, and the expansion of the direction field $[\ell^a]$ all vanish for *any* choice of normalization. Therefore, we can not use these fields to pick out a preferred ℓ^a. However, it turns out that the expansion $\Theta_{(n)}$ of n^a *is* sensitive to its normalization. Furthermore, in static solutions, $\Theta_{(n)}$ is determined entirely by the intrinsic parameters of the horizon. Therefore, it is natural to require that $\Theta_{(n)}$ be the same function of the parameters on *any* isolated horizon. Although it is not apriori obvious, the available rescaling freedom in the choice of n in fact suffices to meet this requirement on *any* isolated horizon. Furthermore, the condition *uniquely* picks out a vector field n^a from the equivalence class $[n^a]$. Having a preferred n^a at our disposal, using the standard normalization $\ell \cdot n = -1$ we can then select an ℓ^a from the equivalence class $[\ell^a]$ uniquely. Finally, we define surface gravity κ to be the acceleration of this 'properly normalized' ℓ^a; i.e., we set $\ell^a \nabla_a \ell^b = \kappa \ell^b$ On Δ.

By construction, κ, so defined, yields the 'correct' surface gravity in the six parameter family of static, dilatonic black-holes. However, the key question is: Do the zeroth and first laws hold for general isolated horizons? This is a key test of our strategy of defining κ in

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[2]In standard treatments, static solutions are parametrized by the ADM mass M, electric and magnetic charges Q and P, dilatonic charge D, cosmological constant Λ and the dilatonic coupling parameter α. Of these, M and D are defined *at infinity*. In the generalized context of isolated horizons, on the other hand, one must use parameters that are intrinsic to Δ. Apriori, it is not obvious that this can be done. It turns out that we can trade M with the area a_Δ of the horizon and D with the value ϕ_Δ of the dilaton field on Δ. Boundary conditions ensure that ϕ_Δ is a constant.

the general case. The answer is in the affirmative.

The zeroth law –constancy of κ on isolated horizons– is established as follows. First, our boundary conditions on $[\ell^a]$ and $[n^a]$ directly imply that κ is constant on each trapped 2-surface. Next, one can show that κ can be expressed in terms of the Weyl curvature component Ψ_2 and the expansion $\Theta_{(n)}$. Finally, the Bianchi identity $\nabla_{[a}R_{bc]de} = 0$, the form of the Ricci tensor component Φ_{11} dictated by our boundary conditions on the matter stress-energy, and our 'normalization condition' on $\Theta_{(n)}$ imply that κ is also constant along the integral curves of ℓ^a. Hence κ is constant on any isolated horizon. To summarize, even though our boundary conditions allow for the presence of radiation arbitrarily close to Δ, they successfully extract enough structure intrinsic to the horizons of static black holes to ensure the validity of the zeroth law. Our derivation brings out the fact that the zeroth law is really local to the horizon: Degrees of freedom of the isolated horizon 'decouple' from excitations present elsewhere in space-time.

To establish the first law, one must first introduce the notion of mass M_Δ of the isolated horizon. The idea is to define M_Δ using the Hamiltonian framework. For this, one needs a well-defined action principle. Fortunately, even though the boundary conditions were designed only to capture the notion of an isolated horizon in a quasi-local fashion, they turn out to be well-suited for the variational principle. However, just as one must add a suitable boundary term at infinity to the Einstein-Hilbert action to make it differentiable in the asymptotically flat context, we must now add another boundary term at Δ. Somewhat surprisingly, the new boundary term turns out to be the well-known Chern-Simons action (for the self-dual connection). This specific form is not important to classical considerations. However, it plays a key role in the quantization procedure. The boundary term at Δ is different from that at infinity. Therefore one can not simultaneously absorb both terms in the bulk integral using Stokes' theorem. Finally, to obtain a well-defined variational principle for the Maxwell part of the action, one needs a partial gauge fixing at Δ. One can follow a procedure similar to the one given above for fixing the rescaling freedom in n^a and ℓ^a. It turns out that, not only does this strategy make the Maxwell action differentiable, but it also uniquely fixes the scalar potential Φ at the horizon.

Having the action at one's disposal, one can pass to the Hamiltonian framework.[3] Now, it turns out that the symplectic structure has, in addition to the standard bulk term, a surface term at Δ. The surface term is inherited from the Chern-Simons term in the action and is therefore precisely the Chern-Simons symplectic structure with a specific coefficient (i.e., in the language of the Chern-Simons theory, a specific value of the 'level' k). The presence of a surface term in the symplectic structure is somewhat unusual; for example, the boundary term at infinity in the action does *not* induce a boundary term in the symplectic structure.

The Hamiltonian consists of a bulk integral and two surface integrals, one at infinity and one at Δ. The presence of two surface integrals is not surprising; for example one encounters it even in the absence of an internal boundary, if the space-times under consideration have

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[3]This passage turns out not to be as straightforward as one might have imagined because there are subtle differences between the variational principles that lead to the Lagrangian and Hamiltonian equations of motion. See [13].

two asymptotic regions. As usual, the bulk term is a linear combination of constraints and the boundary term at infinity is the ADM energy. Using several examples as motivation, we interpret the surface integral at the horizon as the horizon mass M_Δ [13]. This interpretation is supported by the following result: If the isolated horizon extends to future time-like infinity i^+, under suitable assumptions one can show that M_Δ is equal to the future limit, along $\mathcal{I}^+$, of the Bondi mass. Finally, note that M_Δ is *not* a fundamental, independent attribute of the isolated horizon; it is a function of the area a_Δ and charges Q_Δ, P_Δ, ϕ_Δ which are regarded as the fundamental parameters.

Thus, we can now assign to any isolated horizon, an area a_Δ, a surface gravity κ, an electric potential Φ and a mass M_Δ. The electric charge Q_Δ can be defined using the electromagnetic and dilatonic fields field *at* Δ [14]. All quantities are defined in terms of the local structure at Δ. Therefore, one can now ask: if one moves from *any* space-time in $\mathcal{IH}$ to *any* nearby space-time through a variation δ, how do these quantities vary? An explicit calculation shows:

$$\delta M_\Delta = \frac{1}{8\pi G}\ \kappa\delta a_\Delta + \Phi\delta Q_\Delta\ .$$

(For simplicity, I have restricted myself here to the Einstein-Maxwell case without dilaton.) Thus, the first law of black hole mechanics naturally generalizes to isolated horizons. (As usual, the magnetic charge can be incorporated via the standard duality rotation.) This result provides additional support for our strategy of defining κ, Φ and M_Δ.

In static space-times, the mass M_Δ of the isolated horizon coincides with the ADM mass M defined at infinity. In general, M_Δ is the difference between M and the 'radiative energy' of space-time. However, as in the static case, M_Δ continues to include the energy in the 'Coulombic' fields —i.e., the 'hair'— associated with the charges of the horizon, even though it is defined locally at Δ. This is a subtle property but absolutely essential if the first law is to hold in the form given above. To my knowledge, none of the quasi-local definitions of mass shares this property with M_Δ. Finally, isolated horizons provide an appropriate framework for discussing the 'physical process version' of the first law for processes in which the charge of the black hole changes. The standard strategy of using the ADM mass in place of M_Δ appears to run in to difficulties [13] and, as far as I am aware, this issue was never discussed in the literature in the usual context of context of static event horizons.

C. Quantum geometry in the bulk

In this sub-section, I will make a detour to introduce the basic ideas we need from quantum geometry. For simplicity, I will ignore the presence of boundaries and focus just on the structure in the bulk.

There is a common expectation that the continuum picture of space-time, used in macroscopic physics, would break down at the Planck scale. This expectation has been shown to be correct within a non-perturbative, background independent approach to quantum gravity (see [7] and references therein).[4] The approach is background independent in the sense that,

[4]The necessity of a non-perturbative approach is illustrated by the following simple example.

at the fundamental level, there is neither a classical metric nor any other field to perturb around. One only has a bare manifold and *all* fields, whether they represent geometry or matter, are quantum mechanical from the beginning. Because of the subject matter now under consideration, I will focus on geometry.

Quantum mechanics of geometry has been developed systematically over the last three years and further exploration continues [7]. The emerging theory is expected to play the same role in quantum gravity that differential geometry plays in classical gravity. That is, quantum geometry is not tied to a specific gravitational theory. Rather, it provides a kinematic framework or a language to formulate dynamics in a large class of theories, including general relativity and supergravity. In this framework, the fundamental excitations of gravity/geometry are one-dimensional, rather like 'polymers' and the continuum picture arises only as an approximation involving coarse-graining on semi-classical states. The one dimensional excitations can be thought of as flux lines of area [22]. Roughly, each line assigns to a surface element it crosses one Planck unit of area. More precisely, the area assigned to a surface is obtained by algebraic operations (involving group-representation theory) at points where the flux lines intersect the surface. As is usual in quantum mechanics, quantum states of geometry are represented by elements of a Hilbert space [21]. I will denote it by $\mathcal{H}_{\rm bulk}$. The basic object for spatial Riemannian geometry continues to be the triad, but now represented by an operator(-valued distribution) on $\mathcal{H}_{\rm bulk}$ [22]. All other geometric quantities —such as areas of surfaces and volumes of regions— are constructed from the triad and represented by self-adjoint operators on $\mathcal{H}_{\rm bulk}$. The eigenvalues of all geometric operators are discrete; geometry is thus quantized in the same sense that the energy and angular momentum of the hydrogen atom are quantized [22].

There is however, one subtlety: there is a one-parameter ambiguity in this non-perturbative quantization [23]. The parameter is positive, labeled γ and called the Immirzi parameter. This ambiguity is similar to the θ ambiguity in the quantization of Yang-Mills theories. For all values of γ, one obtains the same classical theory, expressed in different canonical variables. However, quantization leads to a one-parameter family of *inequivalent* representations of the basic operator algebra. In particular, in the sector labeled by γ the spectra of the triad —and hence, all geometric— operators depend on γ through an overall multiplicative factor. Therefore, while the qualitative features of quantum geometry are the same in *all* γ sectors, the precise eigenvalues of geometric operators vary from one sector to another. The γ-dependence itself is simple —effectively, Newton's constant G is replaced by γG in the γ-sector. Nonetheless, to obtain unique predictions, it must be eliminated and this requires an additional input. Note however that since the ambiguity involves a single parameter, as with the θ ambiguity in QCD, one judiciously chosen experiment would suffice to eliminate it. Thus, for example, if we could measure the quantum of area , i.e., smallest

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The energy levels of a harmonic oscillator are discrete. However, it would be difficult to see this fundamental discreteness if one were to solve the problem perturbatively, starting from the Hamiltonian of a free particle. Similarly, if one *begins* with a continuum background geometry and then tries to incorporate the quantum effects perturbatively, it would be difficult to unravel discreteness in the spectra of geometric operators such as areas of surfaces or volumes of regions.

non-zero value that area of any surface can have, we would know which value of γ is realized in Nature. Any further experiment would then be a test of the theory. Of course, it is not obvious how to devise a feasible experiment to measure the area quantum directly. However, we will see that it is possible to use black hole thermodynamics to introduce suitable thought experiments. One of them can determine the value of γ and the other can then serve as consistency checks.

D. Quantum geometry of horizon and entropy

Ideas introduced in the last three sub-sections were combined and further developed to systematically analyze the quantum geometry of isolated horizons and calculate their statistical mechanical entropy in [11,16,17]. (For earlier work, see [24,25].) In this discussion, one is interested in space-times with an isolated horizon with *fixed* values a_o, Q_o and ϕ_o of the intrinsic horizon parameters, the area, the electric charge, and the value of the dilaton field.

The presence of an isolated horizon Δ manifests itself in the classical theory through boundary conditions. As usual, we can use some of the boundary conditions to eliminate certain gauge degrees of freedom at Δ. The remaining degrees of freedom are coded in an Abelian connection V defined intrinsically on Δ. V is constructed from the self-dual spin connection in the bulk. It is interesting to note that there are *no* surface degrees of freedom associated with matter: Given the intrinsic parameters of the horizon, boundary conditions imply that matter fields defined intrinsically on Δ can be completely expressed in terms of geometrical (i.e., gravitational) fields at Δ. One can also see this feature in the symplectic structure. While the gravitational symplectic structure acquires a surface term at Δ, matter symplectic structures do not. We will see that this feature provides a simple explanation of the fact that, among the set of intrinsic parameters natural to isolated horizons, entropy depends only on area.

Of particular interest to the present Hamiltonian approach is the pull-back of V to the 2-sphere S_Δ (orthogonal to ℓ^a and n^a) at which the space-like 3-surfaces M used in the phase space construction intersect Δ. (See figure 1(a).) This pull-back —which I will also denote by V for simplicity— is precisely the $U(1)$ spin-connection of the 2-sphere S_Δ. Not surprisingly, the Chern-Simons symplectic structure for the non-Abelian self-dual connection that I referred to in Section III B can be re-expressed in terms of V. The result is unexpectedly simple [11]: the surface term in the total symplectic structure is now just the Chern-Simons symplectic structure for the *Abelian* connection V! The only remaining boundary condition relates the curvature $F = dV$ of V to the triad vectors. This condition is taken over as an operator equation. Thus, in the quantum theory, neither the intrinsic geometry nor the curvature of the horizon are frozen; neither is a classical field. Each is allowed to undergo quantum fluctuations but because of the operator equation relating them, they have to fluctuate in tandem.

To obtain the quantum description in presence of isolated horizons, therefore, one begins with a fiducial Hilbert space $\mathcal{H} = \mathcal{H}_{\rm bulk} \otimes \mathcal{H}_{\rm surface}$ where $\mathcal{H}_{\rm bulk}$ is the Hilbert space associated with the bulk polymer geometry and $\mathcal{H}_{\rm surface}$ is the Chern-Simons Hilbert space for the connection V.[5] The quantum boundary condition says that only those states in $\mathcal{H}$ are allowed for which there is a precise intertwining between the bulk and the surface parts. However, because the required intertwining is 'rigid', apriori it is not clear that the quantum boundary conditions would admit *any* solutions at all. For solutions to exist, there has to be a very delicate matching between certain quantities on $\mathcal{H}_{\rm bulk}$ calculated from the bulk quantum geometry and certain quantities on $\mathcal{H}_{\rm surface}$ calculated from the Chern-Simons theory. The precise numerical coefficients in the surface calculation depend on the numerical factor in front of the surface term in the symplectic structure (i.e., on the Chern-Simons level k) which is itself determined in the classical theory by the coefficient in front of the Einstein-Hilbert action and our classical boundary conditions. Thus, the existence of a coherent quantum theory of isolated horizons requires that the three corner stones —classical general relativity, quantum mechanics of geometry and Chern-Simons theory— be united harmoniously. Not only should the three conceptual frameworks fit together seamlessly but certain *numerical coefficients*, calculated independently within each framework, have to match delicately. Fortunately, these delicate constraints are met and the quantum boundary conditions admit a sufficient number of solutions.

Because we have fixed the intrinsic horizon parameters, is is natural to construct a micro-canonical ensemble from eigenstates of the corresponding operators with eigenvalues in the range $(q_o - \delta q, q_o + \delta q)$ where δq is very small compared to the fixed value q_o of the intrinsic parameters. Since there are no surface degrees of freedom associated with matter fields, let us focus on area, the only gravitational parameter available to us. Then, we only have to consider those states in $\mathcal{H}_{\rm bulk}$ whose polymer excitations intersect S_Δ in such a way that they endow it with an area in the range $(a_o - \delta a, a_o + \delta a)$ where δa is of the order of $\ell_{\rm Pl}^2$ (with $\ell_{\rm Pl}$, the Planck length). Denote by $\mathcal{P}$ the set of punctures that any one of these polymer states makes on S_Δ, each puncture being labeled by the eigenvalue of the area operator at that puncture. Given such a bulk state, the quantum boundary condition tells us that only those Chern-Simons surface states are allowed for which the curvature is concentrated at punctures and the range of allowed value of the curvature at each puncture is dictated by the area eigenvalue at that puncture. Thus, for each $\mathcal{P}$, the quantum boundary condition picks out a sub-space $\mathcal{H}_{\rm surface}^{\mathcal{P}}$ of the surface Hilbert space $\mathcal{H}_{\rm surface}$. Thus, the quantum geometry of the isolated horizon is effectively described by states in

$$\mathcal{H}_{\rm surface}^{\rm phys} = \bigoplus_{\mathcal{P}} \mathcal{H}_{\rm surface}^{\mathcal{P}}$$

as $\mathcal{P}$ runs over all possible punctures and area-labels at each puncture, compatible with the requirement that the total area assigned to S_Δ lie in the given range.

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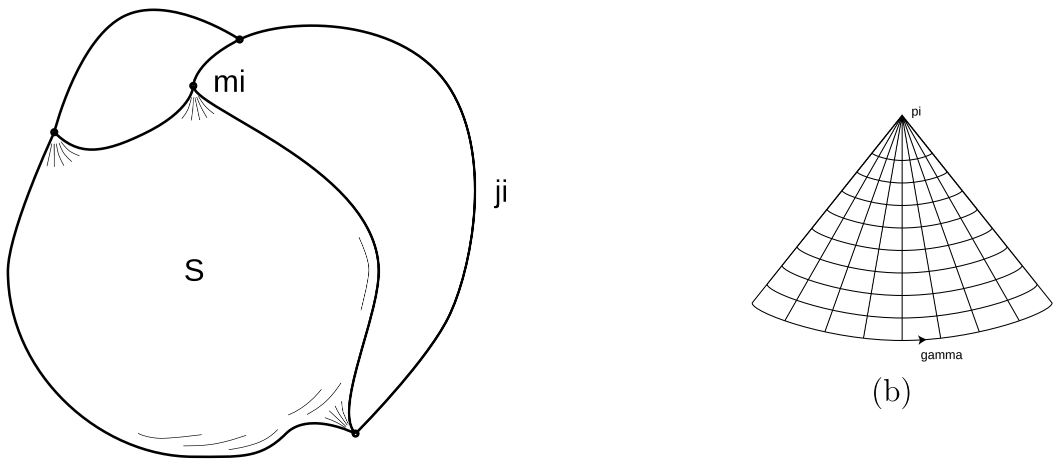
[5]In the classical theory, all fields are smooth, whence the value of any field in the bulk determines its value on Δ by continuity. In quantum theory, by contrast, the measure is concentrated on generalized fields which can be arbitrarily discontinuous, whence surface states are no longer determined by bulk states. A compatibility relation does exist but it is introduced by the quantum boundary condition. It ensures that the total state is invariant under the permissible internal rotations of triads.



(a) (b)

FIG. 3. (a) Quantum geometry around an isolated horizon. The i-th polymer excitation of the bulk geometry carries a 1/2-integer label j_i. Upon puncturing the horizon 2-sphere S_Δ, it induces $8\pi\gamma\sqrt{j_i(j_i+1)}$ Planck units of area. At each puncture, in the intrinsic geometry of S_Δ, there is a deficit angle of $2\pi m_i/k$, where m_i is a 1/2-integer in the interval $[-j_i, j_i]$ and k the 'level' of the Chern-Simons theory. (b) Magnified view of a puncture p_i. The holonomy of the $U(1)$ connection V around a loop γ surrounding any puncture p_i determines the deficit angle at p_i. Each deficit angle is quantized and they add up to 2π.

One can visualize this quantum geometry as follows. Given any one state in $\mathcal{H}^{\mathcal{P}}_{\rm surface}$, the connections V are flat everywhere except at the punctures and the holonomy around each puncture is fixed. Using the classical interpretation of V as the metric compatible spin connection on S_Δ we conclude that, in quantum theory, the intrinsic geometry of the horizon is flat except at the punctures. At each puncture, there is a deficit angle, whose value is determined by the holonomy of V around that puncture. Since each puncture corresponds to a polymer excitation in the bulk, polymer lines can be thought of as 'pulling' on the horizon, thereby producing deficit angles in an otherwise flat geometry (see figure 3). Each deficit angle is quantized and the angles add up to 2π as in a discretized model of a 2-sphere geometry. Thus, the quantum geometry of an isolated horizon is quite different from its smooth classical geometry. In addition, of course, each polymer line endows the horizon with a small amount of area and these area elements add up to provide the horizon with total area in the range $(a_0 - \delta a, a_o + \delta a)$. Thus, one can intuitively picture the quantum horizon as the surface of a large, water-filled balloon which is suspended with a very large number of wires, each exerting a small tug on the surface at the point of contact and giving rise to a 'conical singularity' in the geometry.

Finally, one can calculate the entropy of the quantum micro-canonical ensemble. We are not interested in the *full* Hilbert space since the 'bulk-part' includes, e.g., states of gravitational radiation and matter fields far away from Δ. Rather, we wish to consider only the states of the isolated horizon Δ itself. Therefore, we are led to trace over the 'bulk states' to construct a density matrix $\rho_{\rm IH}$ describing a maximum-entropy mixture of surface states for which the intrinsic parameters lie in the given range. The statistical mechanical entropy is then given by $S = -\mathrm{Tr}\,\rho_{\rm IH}\ln\rho_{\rm IH}$. As usual, the trace can be obtained simply by counting states, i.e., by computing the dimension $\mathcal{N}$ of $\mathcal{H}^{\rm phys}_{\rm surface}$. We have:

$$\mathcal{N} = \exp\,(\frac{\gamma_o}{\gamma}\,\frac{a_o}{4\ell^2_{\rm Pl}}) \qquad \text{where} \qquad \gamma_o = \frac{\ln 2}{\pi\sqrt{3}}$$

Thus, the number of micro-states does go exponentially as area. This is a non-trivial result. For example if, as in the early treatments, one ignores boundary conditions and the Chern-Simons term in the symplectic structure and does a simple minded counting, one finds that the exponent in $\mathcal{N}$ is proportional to $\sqrt{a_o}$. However, our numerical coefficient in front of the exponent depends on the Immirzi parameter γ. The appearance of γ can be traced back directly to the fact that, in the γ-sector of the theory, the area eigenvalues are proportional to γ. Thus, because of the quantization ambiguity, the γ-dependence of $\mathcal{N}$ is inevitable.

We can now adopt the following 'phenomenological' viewpoint. In the infinite dimensional space $\mathcal{IH}$, one can fix one space-time admitting isolated horizon, say the Schwarzschild space-time with mass $M_o >> M_{\rm Pl}$, (or, the de Sitter space-time with the cosmological constant $\Lambda_o << 1/\ell^2_{\rm Pl}$). For agreement with semi-classical considerations, in these cases, entropy should be given by $S = (a_o/4\ell^2_{\rm Pl})$ which can happen only in the sector $\gamma = \gamma_o$ of the theory. The theory is now completely determined and we can go ahead and calculate the entropy of any other isolated horizon in *this* theory. Clearly, we obtain:

$$S_{\rm IH} = \frac{1}{4}\,\frac{a_o}{\ell^2_{\rm Pl}}$$

for *all* isolated horizons. Furthermore, in this γ-sector, the statistical mechanical temperature of any isolated horizon is given by Hawking's semi-classical value $\kappa\hbar/2\pi$ [8,24]. Thus, we can do one thought experiment —observe the temperature of a large black black hole from far away— to eliminate the Immirzi ambiguity and fix the theory. This theory then predicts the correct entropy and temperature for all isolated horizons in $\mathcal{IH}$ with $a_o >> \ell^2_{\rm Pl}$.

The technical reason behind this universality is trivial. However, the conceptual argument is not because it is quite non-trivial that $\mathcal{N}$ depends only on the area and not on values of other charges. Furthermore, the space $\mathcal{IH}$ is infinite dimensional and it is not apriori obvious that one should be able to give a statistical mechanical account of entropy of *all* isolated horizons in one go. Indeed, values of fields such as Ψ_4 and ϕ_2 can be vary from one isolated horizon to another even when they have same intrinsic parameters. This freedom could well have introduced obstructions, making quantization and entropy calculation impossible. That this does not happen is related to but independent of the fact that this feature did not prevent us from extending the laws of mechanics from static event horizons to general isolated horizons.

I will conclude this sub-section with two remarks.

i) In this approach, we began with the sector of general relativity admitting isolated horizons and then quantized that sector. Therefore, ours is an 'effective' description. In a fundamental description, one would begin with the full quantum theory and isolate in it the sector corresponding to quantum horizons. Since the notion of horizon is deeply tied to classical geometry, at the present stage of our understanding, this goal appears to be out of reach in all approaches to quantum gravity. However, for thermodynamic considerations of large horizons, the effective description should be sufficient.

ii) The notion of entropy used here has two important features. First, in this framework, the notion is not an abstract property of the space-time as a whole but depends on the division of space-time in to an exterior and an interior. Operationally, it is tied to the class of observers who live in the exterior region for whom the isolated horizon is a *physical* boundary that separates the part of the space-time they can access from the part they can not. (This is in sharp contrast to early work which focussed on the interior.) This point is especially transparent in the case of cosmological horizons in de Sitter space-time since that space-time does not admit an invariantly defined division. The second feature is that, although there is 'observer dependence' in this sense, the entropy does *not* refer to the degrees of freedom in the interior. Indeed, nowhere in our calculation did we analyze the states associated with the interior. Rather, our entropy refers to the micro-states of the boundary itself which are compatible with the macroscopic constraints on the area and charges of the horizon; it counts the physical micro-states which can interact with the outside world, not disconnected from it.

IV. DISCUSSION

Perhaps the most pleasing aspect of this analysis is the existence of a single framework to encompass diverse ideas at the interface of general relativity, quantum theory and statistical mechanics. In the classical domain, this framework generalizes laws of black hole mechanics to physically more realistic situations. At the quantum level, it provides a detailed description of the quantum geometry of horizons and leads to a statistical mechanical calculation of entropy. In both domains, the notion of isolated horizons provides an unifying arena enabling us to handle different types of situations —e.g., black holes and cosmological horizons— in a single stroke. In the classical theory, the same line of reasoning allows one to establish the zeroth and first laws for *all* isolated horizons. Similarly, in the quantum theory, a single procedure leads one to quantum geometry and entropy of *all* isolated horizons. By contrast, in other approaches, fully quantum mechanical treatments seem to be available only for stationary black holes. Indeed, to my knowledge, even in the static case, a complete statistical mechanical calculation of the entropy of cosmological horizons has not been available. Finally, our extension of the standard *Killing* horizon framework sheds new light on a number of issues, particularly the notion of mass of associated to an horizon and the physical process version of the first law [13].

However, the framework presented here is far from being complete and provides promising avenues for future work. First, while some of the motivation behind our approach is similar to the considerations that led to the interesting series of papers by Brown and York [10], not much is known about the relation between the two frameworks. It would be interesting to explore this relation, and more generally, to relate the isolated horizon framework to the semi-classical ideas based on Euclidean gravity. Second, while the understanding of the micro-states of an isolated horizon is fairly deep by now, work on a quantum gravity derivation of the Hawking radiation is still in a preliminary stage. Using general arguments based on Einstein's A and B coefficients [1] and the known micro-states of an isolated horizon, one can argue [18] that the envelope of the line spectrum emitted by a black hole should be thermal. However, further work is necessary to make sure that the details are correct.

For the laws of mechanics and the entropy calculation, the obvious open problem is the extension to incorporate non-zero angular momentum. Recently, Jerzy Lewandowski has performed an exhaustive analysis of the geometrical structure of general isolated horizons and streamlined the necessary background material. further recent work in collaboration with him and with Chris Beetle and Steve Fairhurst has led to a generalization of boundary conditions to incorporate rotation (as well as distortion in absence of rotation) and a proof of the zeroth law in the general context. Construction of the corresponding Hamiltonian framework is now under way. The extension of the entropy calculation, on the other hand, may turn out to be trickier for it may well require a new technical insight. On a long range, the outstanding challenge is to obtain a deeper understanding of the Immirzi ambiguity and the associated issue of renormalization of Newton's constant. For any value of γ, one obtains the 'correct' classical limit. However, as far as black hole thermodynamics is concerned, it is only for $\gamma = \gamma_o$ that one seems to obtain agreement with quantum field theory in curved space-times. Is this value of γ robust? Can one make further semi-classical checks? A pre-requisite for this investigation is a better handle on the issue of semi-classical states. A major effort will soon be devoted to this issue.

Let me conclude with a comparison between the entropy calculation in this approach and those performed in string theory. First, there are some obvious differences. In the present approach, one begins with the sector of the classical theory containing space-times with isolated horizons and then proceeds with quantization. Consequently, one can keep track of the physical, curved geometry. In particular, as required by physical considerations, the micro-states which account for entropy can interact with the physical exterior of the black hole. In string theory, by contrast, actual calculations are generally performed in flat space and non-renormalization arguments and/or duality conjectures are then invoked to argue that the results so obtained refer to macroscopic black holes. Therefore, relation to the curved space geometry and physical meaning of the degrees of freedom which account for entropy is rather obscure. More generally, lack of direct contact with physical space-time can also lead to practical difficulties while dealing with other macroscopic situations. For example, in string theory, it may be difficult to account for the entropy normally associated with de Sitter horizons. On the other hand, in the study of genuinely quantum, Planck size black holes, this 'distance' from the curved space-time geometry may turn out to be a blessing, as classical curved geometry will not be an appropriate tool to discuss physics in these situations. In particular, a description which is far removed from space-time pictures may be better suited in the discussion of the last stages of Hawking evaporation and the associated issue of 'information loss'.

The calculations based on string theory have been carried out in a number of space-time dimensions while the approach presented here is directly applicable only to four dimensions. An extension of the underlying non-perturbative framework to higher dimensions was recently proposed by Freidel, Krasnov and Puzzio but a systematic development of quantum geometry has not yet been undertaken. Also, our quantization procedure has an inherent γ-ambiguity which trickles down to the entropy calculation. By contrast, calculations in string theory are free of this problem. On the other hand, most detailed calculations in string theory have been carried out only for (a sub-class of) extremal or near-extremal black holes. While these black holes are especially simple to deal with mathematically, unfortunately, they are not of direct relevance to astrophysics, i.e., to the physical world we live in. More recently, using the Maldecena conjecture, stringy calculations have been extended to non-extremal black holes with $R_{\rm Sch}^2 >> 1/\Lambda$, where $R_{\rm Sch}$ is the Schwarzschild radius. However, the numerical coefficient in front of the entropy turns out to be incorrect and it is not yet clear whether inclusion of non-Abelian interactions, which are ignored in the current calculations, would restore the numerical coefficient to its correct value. Furthermore, it appears that a qualitatively new strategy may be needed to go beyond the $R_{\rm Sch}^2 >> 1/\Lambda$ approximation. Finally, as in other results based on the Maldecena conjecture, the underlying boundary conditions at infinity are quite unphysical since the radius of the compactified dimensions is required to equal the cosmological radius even near infinity. Hence the relevance of these mathematically striking results to our physical world remains unclear. In the current approach, by contrast, ordinary, astrophysical black holes in the physical, four space-time dimensions are included from the beginning.

In spite of this differences, there are some striking similarities. Our polymer excitations resemble stings. Our horizon looks like a 'gravitational 2-brane'. Our polymer excitations ending on the horizon, depicted in figure 3, closely resemble strings with end points on a membrane. As in string theory, our '2-brane' carries a natural gauge field. Furthermore, the horizon degrees of freedom arise from this gauge field. These similarities seem astonishing. However, a closer look brings out a number of differences as well. In particular, being horizon, our '2-brane' has a direct interpretation in terms of the curved *space-time geometry* and our $U(1)$ connection is the *gravitational* spin-connection on the horizon. Nonetheless, it may well be that, when quantum gravity is understood at a deeper level, it will reveal that the striking similarities are not accidental, i.e., that the two descriptions are in fact closely related.

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REFERENCES

Classics

[1] J.D. Bekenstein, Black holes and entropy, *Phys. Rev.* **D7** 2333-2346 (1973).
J.D. Bekenstein, Generalized second law of thermodynamics in black hole physics, *Phys. Rev.* **D9** 3292-3300 (1974).
J. D. Bekenstein and A. Meisels, Einstein A and B coefficients for a black hole, *Phys. Rev.* **D15**, 2775-2781 (1977).

[2] J.W. Bardeen, B. Carter and S.W. Hawking, The four laws of black hole mechanics, *Commun. Math. Phys.* **31** 161-170 (1973).
B. Carter, Black hole equilibrium states, In *Black Holes*, ed. B. DeWitt and C. DeWitt, Gordon and Breach, New York, 1973.

[3] S.W. Hawking, Particle creation by black holes, *Commun. Math. Phys.* **43** 199-220 (1975).

[4] G. Gibbons and S.W. Hawking, Cosmological event horizons, thermodynamics, and particle creation, *Phys. Rev.* **D15** 2738-2751 (1977).

Reviews

[5] M. Heusler, *Black Hole Uniqueness Theorems*, Cambridge University Press, Cambridge, 1996.

[6] R.M. Wald, *Quantum Field Theory in Curved Space-Time and Black Hole Thermodynamics*, University of Chicago Press, Chicago, 1994.

[7] A. Ashtekar, Quantum mechanics of geometry, In *The Narlikar Festschrift*, ed. N. Dadhich and A. Kembhavi, 1999, also available as gr-qc/990123.

[8] A. Ashtekar, K. Krasnov. Quantum geometry and black holes, In *Black holes, gravitational radiation and the Universe*, ed. B. Bhawal and B.R. Iyer, Kluwer, Dordrecht, 149-170 (1998); also available as gr-qc/9804039.

[9] G. Horowitz, Quantum states of black holes, In *Black holes and relativistic stars*, ed. R.M. Wald, U. Chicago press, Chicago, 1998.

[10] J. D. Brown and J. W. York, The path integral formulation of gravitational thermodynamics, available as gr-qc/9405024.

Main papers where details can be found

[11] A. Ashtekar, A. Corichi and K. Krasnov, Isolated horizons: the classical phase space, *Adv. Theor. Math. Phys.* in press; also available as gr-qc/9905089.

[12] A. Ashtekar, C. Beetle and S. Fairhurst, Isolated horizons: a generalization of black hole mechanics, *Class. Quantum Grav.* **16** L1-L7 (1999).

[13] A. Ashtekar, C. Beetle and S. Fairhurst, Mechanics of isolated horizons, *Class. Quantum Grav.* (in press), also available as gr-qc/9907068.

[14] A. Ashtekar and A. Corichi, Laws governing dilatonic isolated horizons, Submitted to *Class. Quantum Grav.*, also available as gr-qc/9910068.

[15] J. Lewandowski, Space-times admitting isolated horizons, Submitted to *Class. Quantum Grav.*, also available as gr-qc/9907058.

[16] A. Ashtekar, J. Baez, A. Corichi, K. Krasnov, Quantum geometry and black hole entropy, *Phys. Rev. Lett.* **80** 904-907 (1998).

[17] A. Ashtekar, J. Baez, K. Krasnov, Quantum geometry of isolated horizons horizons and black hole entropy, Pre-print.

[18] K. Krasnov, Quantum geometry and thermal radiation form black Holes, *Class. Quantum. Grav.* **16**, 563 (1999).

Related references: Classical black holes

[19] S. Hayward, General laws of black hole dynamics, *Phys. Rev.* **D49** 6467-6474 (1994).
S. Hayward, Spin coefficient form of the new laws of black hole dynamics, *Class. Quant. Grav.* **11** 3025-3036 (1994).
S. Hayward, Relativistic thermodynamics, gr-qc/9803007.

[20] D. Sudarski and R.M. Wald, Extrema of mass, stationarity and staticity, and solutions to the Einstein-Yang-Mills equations, *Phys. Rev.* **D46** 1453-1474 (1992).
R.M. Wald, Black hole entropy is Noether charge, *Phys. Rev.* **D48** 3427-3431 (1993).

Related references: Quantum Geometry

[21] A. Ashtekar and J. Lewandowski, Representation theory of analytic holonomy $C^\star$-algebras, In *Knots and quantum gravity*, ed. J. Baez, Oxford University Press, Oxford, 1994.
A. Ashtekar and J. Lewandowski, Projective techniques and functional integration for gauge theories, *J. Math. Phys.* **36**, 2170-2191 (1995).
J. Baez, Diffeomorphism invariant generalized measures on the space of connections modulo gauge transformations, In *The Proceedings of the conference on quantum topology*, ed. D. Yetter, World Scientific, Singapore, 1994.
J. Baez, Generalized measures in gauge theory, *Lett. Math. Phys.* **31**, 213-223 (1994).
D. Marolf and J. Mourão, On the support of the Ashtekar-Lewandowski measure, *Commun. Math. Phys.* **170**, 583-605 (1995).
C. Rovelli and L. Smolin, Spin networks and quantum gravity, *Phys. Rev.* **D52**, 5743-5759 (1995).
J. Baez, Spin-networks in gauge theory, *Adv. Math.* **117**, 253-272 (1996).
J. Baez, Spin networks in non-perturbative quantum gravity, In *The Interface of Knots and Physics*, ed. L. Kauffman, American Mathematical Society, Providence, 1996, pp. 167-203.

[22] A. Ashtekar and J. Lewandowski, Differential geometry on the space of connections using projective techniques, *J. Geo. Phys.* **17**, 191-230 (1995).
C. Rovelli and L. Smolin, Area and volume in quantum gravity, *Nucl. Phys.* **B442**,
A. Ashtekar and J. Lewandowski, Quantum theory of geometry I: Area operators, *Class. Quant. Grav.* **14**, A55-A81 (1997).
K. Krasnov, On the constant that fixes the area spectrum in canonical quantum gravity, *Class. Quant. Grav.* **15**, L1-L4 (1998).

[23] F. Barbero, From Euclidean to Lorentzian general relativity: the real way, *Phys. Rev.* **D54**, 1492-1499 (1996).
G. Immirzi, Quantum Gravity and Regge Calculus, *Nucl. Phys. Proc. Suppl.* **57**, 65-72 (1997).

R. Gambini, O. Obregón and J. Pullin, Yang-Mills analogs of the Immirzi ambiguity, *Phys. Rev.* **D59**, 047505 (1998).
C. Rovelli, T. Thiemann, The Immirzi parameter in quantum general relativity, *Phys. Rev.* **D57**, 1009-1014 (1998).

Related references: Quantum black holes

[24] K. Krasnov, Geometrical entropy from loop quantum gravity, *Phys. Rev.* **D55** 3505-3513 (1997).
K. Krasnov, On statistical mechanics of Schwarzschild black holes, *Gen. Rel. Grav.* **30**, No. 1, 53-68 (1998).

[25] C. Rovelli, Black hole entropy from loop quantum gravity, *Phys. Rev. Lett.* **14** 3288-3291 (1996).
C. Rovelli, Loop quantum gravity and black hole physics, *Helv. Phys. Acta.* **69** 582-611 (1996).
M. Barreira, M. Carfora, C. Rovelli, Physics with non-perturbative quantum gravity: radiation from a quantum black hole, *Gen. Rel. and Grav.* **28**, 1293-1299 (1996).

[26] R.K. Kaul and P. Majumdar, Quantum black hole entropy, *Phys. Lett.* **B439** 267-270, (1998).

[27] J. Wheeler, It from bits and quantum gravity, In *Sakharov Memorial Lectures on Physics*, ed. L. Keldysh and V. Feinberg, Nova Science, New York, 1992, Vol. 2.

[28] A. Strominger and C. Vafa, Microscopic Origin of the Bekenstein-Hawking Entropy, *Phys. Lett.* **B379**, 99-104, (1996)
J. Maldacena and A. Strominger, Statistical entropy of four-dimensional extremal black holes, *Phys. Rev. Lett.* **77** 428-429 (1996).